1. Is it possible that an event is independent of itself? If so, when?

Sol)

No, it is not possible for an event to be independent of itself. By definition, for two events A and B to be independent, the probability of their joint occurrence (P(A ∩ B)) should be equal to the product of their individual probabilities (P(A) \* P(B)). However, if A is the same event as B, then P(A ∩ B) = P(A), which implies that P(A) should be equal to P(A) \* P(A). This is only true when P(A) is either 0 or 1, which means that A is a certain event or an impossible event.

1. Is it always true that if A and B are independent events, then Ac and Bc are independent events? Show that it is, or give a counterexample.

Sol)

It is not always true that if A and B are independent events, then Ac and Bc are independent events. Here's a counterexample:

Let A and B be two events such that P(A) = 0.5 and P(B) = 0.5. Since A and B are independent, we have P(A ∩ B) = P(A) \* P(B) = 0.5 \* 0.5 = 0.25.

Now, consider the complements of A and B. Ac is the complement of A and Bc is the complement of B. Since P(A) = 0.5, P(Ac) = 1 - P(A) = 0.5. Similarly, P(Bc) = 1 - P(B) = 0.5.

If Ac and Bc were independent, we would expect P(Ac ∩ Bc) to be equal to P(Ac) \* P(Bc), which is 0.5 \* 0.5 = 0.25.

However, in our counterexample, let's assume that P(Ac ∩ Bc) = 0.4, which is not equal to 0.25. Therefore, we have a counterexample where A and B are independent events, but Ac and Bc are not independent.